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Effect and Analysis of Induced Thermal Stresses in a Potted Cylindrical Inductor

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Synopsis

Thermal stresses developed during the cooling of a cured potted inductor from higher processing temperatures, resulting from the difference in the thermal expansion coefficients of the ferrite core and the epoxy polymer, cause a drop in inductance. The inductance drop was followed with time and temperature under various conditions. An attempt is made to explain the inductance drop based on experimental observations and theoretical considerations. A mathematical model was constructed to aid the thermal stress analysis at the ferrite core-polymer interface. It is shown that lower-temperature epoxy curing results in greater stresses (inductance drop) than higher-temperature curing.

INTRODUCTION

A common practice in the electrical industry today is to pot electronic components with polymers for protection. The main protection is against mechanical shock and vibration and against hostile atmospheric constituents such as water vapor.

Sometimes, however, the potting process affects certain properties of the embedded electronic components. A case in point is the inductance of pressure-sensitive ferrite inductors. Thermal stresses developed during the cooling of the cured component from higher processing temperatures, due to the mismatch of coefficients of thermal expansion between the embedded inductor and the polymer system under consideration, lead to a drop in inductance. This temperature-dependent change of inductance presents a problem in the applications of high-performance inductors and transformers using ferrite cores.

Results from another investigation[1] indicated that the majority of polymers studied cause drop in inductance after inductor potting, although some exceptions were noted where no change of inductance, before and after inductor potting, was observed.

In the present study, the effect of thermal stresses, developed at the ferrite core-polymer interface during and after the curing of a particular epoxy resin, on a potted inductor was investigated by following inductance with time and temperature. Also, a mathematical model was used for the analysis of the thermal stresses at the ferrite core–polymer interface, and stresses were calculated with the aid of an electronic computer.

EXPERIMENTAL

Bisphenol A epoxy resin cured with diethanolamine adduct was used as potting material for a ferrite core inductor which is pressure sensitive. The "cup"-type cylindrical core, No. 26/16-3B7 Ferroxcube Corp., consisting of two cores held together with Hysol 0151 epoxy cured 2 hr at 85°C, measured $1 \times {}^5/_8$ in. The core, windings and terminals, characterized as experimental system FTD #427, were potted in a plastic cylindrical container having inside dimensions of $1{}^1/_8 \times 1{}^1/_4$ in.

The parameter studied was the inductance *L*. Measurements were made on a General Radio incremental inductance bridge Model 1633-A. A Hewlett-Packard audio oscillator Model 200 CD was used to power the inductance bridge. The oscillator output was set at 10 kHz with 18.5 V peak to peak. The calibration of the inductance bridge was periodically checked using GR 1482-H (10 mH) and 1482-J (20 mH) standard inductors.

Inductance measurements for each sample were made at three different times during the experiments. The first one was before the epoxy was added (blank). In this case the inductor was placed inside the plastic container and heated to the curing temperature. Subsequently, the system was allowed to cool to room temperature, while inductance and temperature readings were taken every minute for 60 min and then at longer intervals for 1 to 2 hr. The inductor was then reheated to curing temperature, and 8 g of epoxy mixture (DP 2786 Part A and DP 2786 Part B, from Conap, Inc., 100 and 15 parts by weight) was added. A copper–Constantan thermocouple, leading to a United Systems Corp. Digitec digital thermometer Model 564-N, was placed into the epoxy for temperature measurements. The sample was kept at the curing temperature for a specified period of time enough to cure the epoxy. No change in inductance was observed during curing. After the prescribed curing time was over, the oven was turned off and the oven door opened. The inductance was followed with temperature and time as the sample cooled to room temperature at 1-min intervals for 1 hr and at various intervals for up to 4 hr after the oven door was opened. The third set of inductance readings was taken about a day later. The sample was again heated to the curing temperature, held at that temperature for about 3 hr, and then allowed to cool to room temperature. The inductance and temperature were recorded in the same manner as in cooling from curing.

Linear coefficients of thermal expansion for the ferrite and epoxy, from room temperature to 100°C, were determined with the du Pont 941 thermomechanical analyzer.

DISCUSSION

A typical behavior of inductance versus time is shown in Figure 1. The initial increase of inductance in the first 3–5 min of cooling, a phenomenon

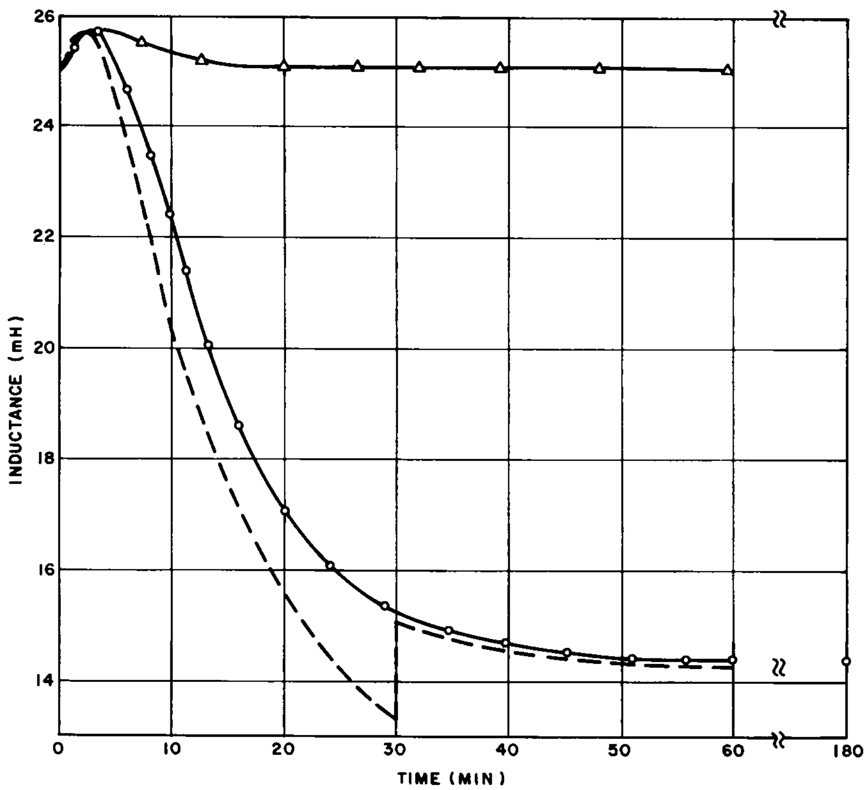

Fig. 1. Inductance drop as a function of time: (Δ) before potting; (O) after potting.

consistent with all of the samples examined in this study, is due to the small stresses developed at the ferrite–polymer interface during this period of time. It has been shown[2] that if the magnetostriction is negative, as in the present case, a small compressive stress will raise the permeability (inductance). However, large stresses lower the permeability (inductance). Consequently, as more and more stress develops with time of cooling, as a result of the mismatch of coefficients of thermal expansion between the ferrite core and the epoxy with temperature, the inductance drops continuously until it reaches a steady-state value when the potted inductor is at room temperature.

Another point for discussion is the rather interesting discrepancy of the inductance drop observed in some experiments between cooling from cure and reheating recooling. In the majority of experiments there was no difference in inductance drop between cooling from cure and reheating recooling after cure, as represented by the solid curve of inductance drop in Figure 1. However, in some experiments, and under exactly the same experimental conditions, the inductance drop followed the dotted curve of Figure 1 during cooling from cure. As can be seen, the inductance decreases until a point in time where it jumps and, for experimental purposes, almost stays there as time of cooling goes on.

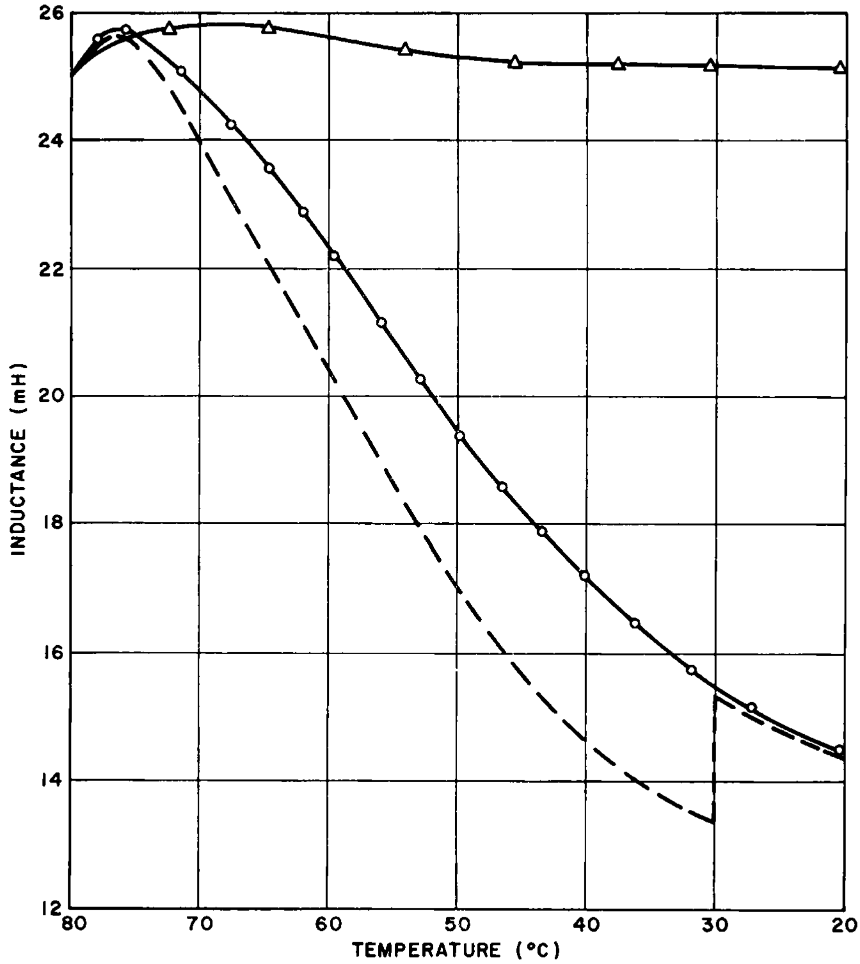

Fig. 2. Inductance drop as a function of temperature: (Δ) before potting; (O) after potting.

When this sample was reheated and recooled, it did follow the solid curve of Figure 1, behaving like a "normal" sample. A possible explanation of this phenomenon could be a glass transition temperature of the polymer at the point of jump[1] associated with stress relaxation (inductance recovery), provided the sample was exhibiting this jump during thermal cycling, since the glass transition phenomenon is reversible with temperature. Since the jump was not reversible and no glass transition temperature could be established at the temperature of the jump by differential thermal analysis (DTA), another explanation should be given. It is true that this is rather difficult, and the only explanation at this time is either a slip between the contacting surfaces or a possible rearrangement of the polymer molecules, at the time of the jump, into a less compact configura-

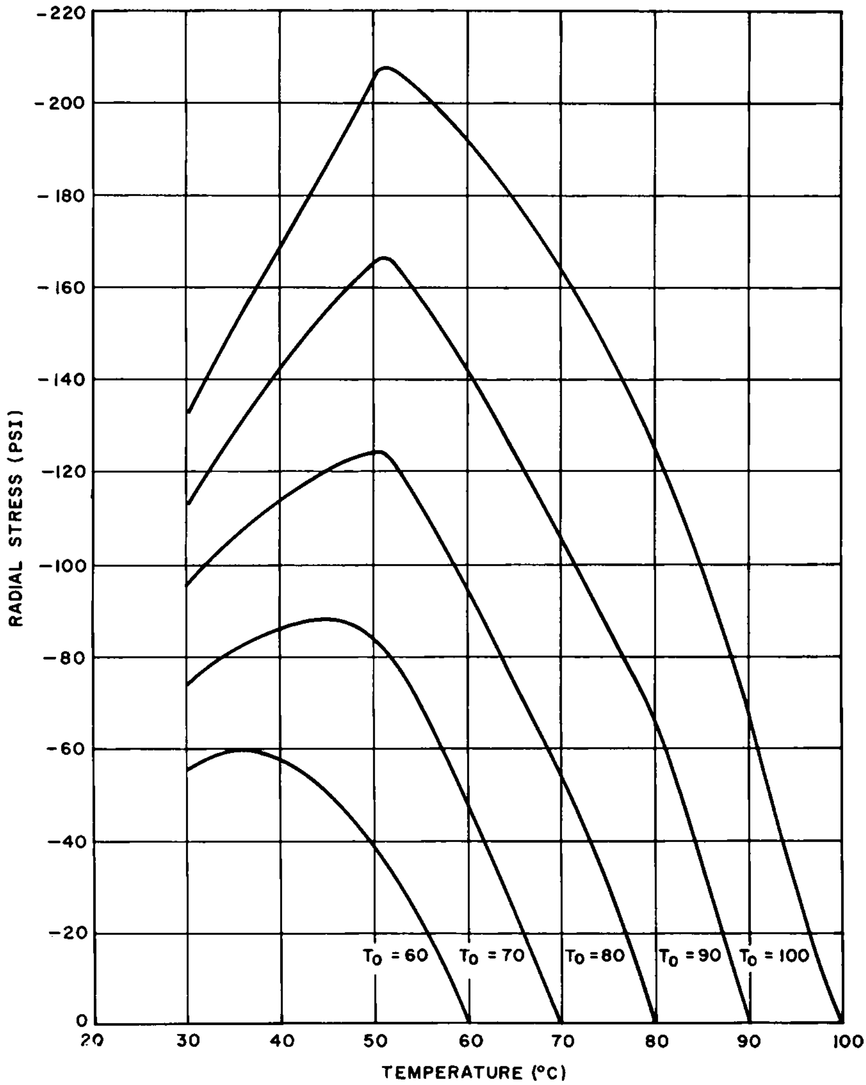

Fig. 3. Radial stress as a function of temperature.

tion on a random probabilistic basis, with the result that the jump is not observed in all of the experiments but in some of them.

Figure 2 represents the inductance versus temperature behavior of a typical inductor. The inductance jump takes place around 30°C, and it appears that this may be the point of maximum stress. To explore this possibility, radial and axial stresses developed at the ferrite core–polymer interface were calculated with the aid of an electronic computer. A mathematical model developed by Poritsky[5] was used (see Appendix) in conjunction with the various parameters listed in Tables I and II. Calcu-

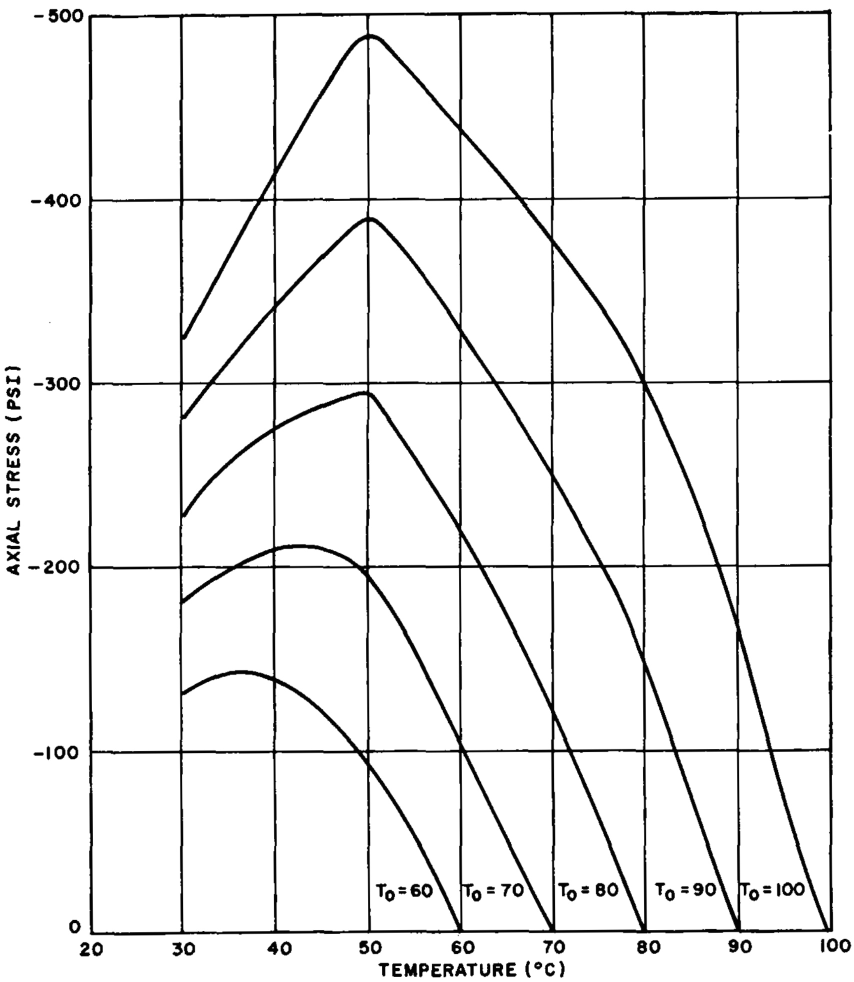

Fig. 4. Axial stress as a function of temperature.

lations were made using eqs. (A-12) of the Appendix. The results for the radial and axial stresses are plotted in Figures 3 and 4, respectively, and indicate that the maximum stresses occur at about 50°C. Thus, the possibility of explaining the jump on the basis of the maximum stress is not valid, unless a large temperature gradient is assumed between the epoxy, where the thermocouple is, and the core, leading to the large temperature lag observed.

Per cent inductance drop as a function of cooling time is shown in Figure 5. As can be seen, curing of the epoxy at lower temperatures (70°C) causes a greater overall inductance drop. This is an unexpected finding. One would expect the opposite to happen, based on the buildup of greater stresses, due to the difference of coefficients of thermal expansion between

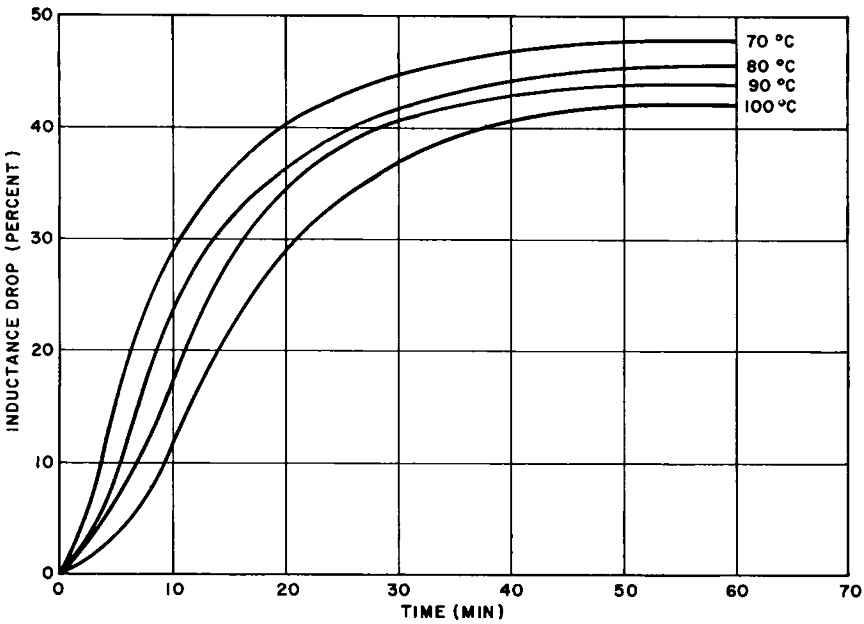

Fig. 5. Per cent inductance drop as a function of time.

the epoxy and the ferrite core, when the cured inductor is cooling from higher processing temperatures (100°C). Another factor which should be taken into account in trying to explain this behavior is the viscoelastic characteristics of the epoxy. It seems that the stresses generated by the epoxy at the ferrite core–polymer interface are a function of the cross-linking density of the epoxy in addition to the thermal expansion characteristics. From the above, it can be concluded that the lower the curing temperature, the harder the epoxy and the higher the curing temperature, the softer the epoxy. This is in agreement with previous studies where it has been shown[3,4] that bisphenol A epoxy cured with various curing agents at lower temperatures was hard and of high density, whereas high-temperature curing resulted in soft, low-density epoxy. Recent experiments[6]

TABLE I
Properties of Ferrite Core and Epoxy

Property	Ferrite	Epoxy
Young's modulus, psi $\times 10^{-6}$	18.4[a]	0.2[b]
Poisson's ratio	0.3[c]	0.33[c]
Coefficient of thermal expansion (in./in. °C) $\times 10^{5}$	1.0[b]	see Table II[b]
Radius, in.	0.504[b]	0.581[b]

[a] Ferroxcube Corp., Saugerties, N. Y.
[b] This study.
[c] Typical value.

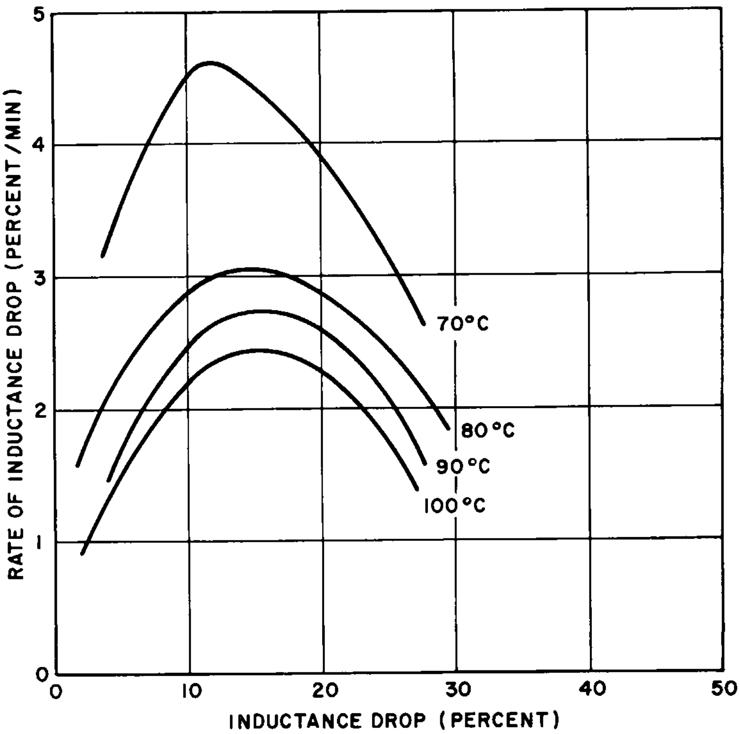

Fig. 6. Rates of inductance drop.

indicate that the higher the temperature of curing, the lower the rate of the reaction after the gel point.

In Figure 6, the rates of inductance drop are plotted against per cent inductance drop. These rates were calculated from the slopes of the per cent inductance drop–time curves with the aid of an electronic computer.

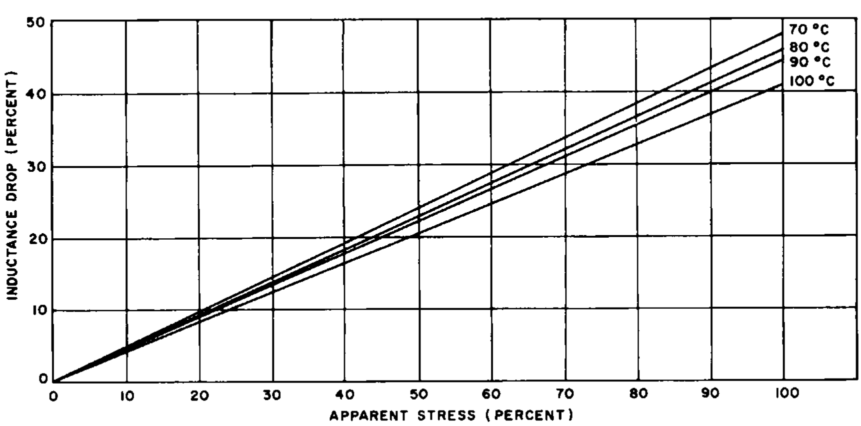

Fig. 7. Per cent inductance drop as a function of apparent stress.

TABLE II
Coefficients of Thermal Expansion of Epoxy at Various Temperatures

Temp., °C	Coefficient of thermal expansion of epoxy (in./in. °C) × 10^5
95	18.9
90	18.8
85	18.5
80	18.2
75	15.8
70	13.7
65	13.4
60	12.9
55	12.1
50	11.7
45	9.9
40	8.4
35	7.3
30	5.9

The data in Figure 6 indicate that distinct maxima were reached at approximately 15% inductance drop. The inverse relationship between curing temperature and rate of inductance drop, as discussed before for the overall inductance drop, is present again.

Per cent inductance drop versus per cent apparent stress is shown in Figure 7. Apparent stress is defined as the ratio of the maximum inductance minus the inductance at time t over the maximum industance minus the minimum inductance. This linear relationship confirms the points discussed before regarding curing temperature and overall inductance drop.

Appendix

Analysis of Thermal Stresses

Equations are derived for the thermal stresses in two concentric joined cylinders due to different coefficients of expansion, on the basis that the material of the cylinder is elastic. Let the radius of the ferrite core be a and the radius of the ferrite core plus the polymer be b, the material of the two cylinders be in contact at a temperature T_0, and the temperature then changed. The problem is to determine what stresses develop in the cylinders at a temperature T if the materials of the two cylinders possess different coefficients of thermal expansion. The solution was taken from Poritsky,[5] after certain corrections were made.

We assume that no plastic flow or thermal creep occurs; that the substances of the cylinders are characterized by conventional elastic constants E and ν (E = Young's modulus, ν = Poisson's ratio); and that these are independent of temperature. We denote by α the coefficients of linear expansion, and distinguish constants and formulas pertaining to the inner cylinder from those relating to the outer one by using subscripts 1 in the former case and 2 in the latter.

Under these conditions, the axial and radial directions are directions of principal stress. Introducing cylindrical coordinates r, θ, z, we shall denote the stress components in the directions of increasing r, θ, and z by σ_r, σ_θ, and σ_z, respectively.

We propose the following expressions for the stresses in the outer cylinder at temperature T:

$$\sigma_r = A_2 + (B_2/r^2),\ \sigma_\theta = A_2 - (B_2/r^2),\ \sigma_z = C_2 \tag{A-1}$$

where A_2, B_2, and C_2 are constants. We also propose similar expressions with subscripts 2 changed to 1 for the inner cylinder, omitting, however, the B_1/r^2 term as this would become infinite at $r = 0$:

$$\sigma_r = A_1,\ \sigma_\theta = A_1,\ \sigma_z = C_1. \tag{A-2}$$

To establish them, we shall prove first that they satisfy the equations of equilibrium. Then we shall find the strains ϵ_r, ϵ_θ, and ϵ_z by applying Young's law and taking the thermal expansions into account; thus:

$$\begin{aligned}\epsilon_r &= (\sigma_r/E) - \nu(\sigma_\theta + \sigma_z)/E + \alpha(T - T_0)\\ \epsilon_\theta &= (\sigma_\theta/E) - \nu(\sigma_r + \sigma_z)/E + \alpha(T - T_0)\\ \epsilon_z &= (\sigma_z/E) - \nu(\sigma_r + \sigma_\theta)/E + \alpha(T - T_0)\end{aligned} \tag{A-3}$$

and then establish the compatibility relations for the strains, namely, the condition that they can be derived from two displacement components u and w:

$$\epsilon_r = \partial u/\partial r,\ \epsilon_\theta = u/r,\ \epsilon_z = \partial w/\partial z \tag{A-4}$$

where u is the radial displacement and w is the axial displacement.

For the axial symmetry, the equations of equilibrium reduce to

$$\partial(r\sigma_r)/\partial r - \sigma_\theta = 0,\ \partial\sigma_z/\partial z = 0.$$

Equations (A-3) now yield (with E, ν, and α replaced by E_2, ν_2, and α_2):

$$\begin{aligned}E_2\epsilon_r &= A_2(1 - \nu_2) + B_2(1 + \nu_2)/r^2 - \nu_2 C_2 + E_2\alpha_2(T - T_0)\\ E_2\epsilon_\theta &= A_2(1 - \nu_2) - B_2(1 + \nu_2)/r^2 - \nu_2 C_2 + E_2\alpha_2(T - T_0)\\ E_2\epsilon_z &= A_2(-2\nu_2) + C_2 + E_2\alpha_2(T - T_0).\end{aligned} \tag{A-5}$$

These are compatible with eqs. (A-4), provided u is given by

$$E_2 u = [A_2(1 - \nu_2) - \nu_2 C_2 + E_2\alpha_2(T - T_0)]r - B_2(1 + \nu_2)/r. \tag{A-6}$$

While the above proof applies to eqs. (A-1) and to the outer cylinder, it can also be made to cover eqs. (A-2) and the inside cylinder by changing the subscripts 2 to 1 and putting $B_1 = 0$.

With (A-1) and (A-2) thus established as possible states of stress, it remains to show that the arbitrary elements left, namely, the constants A_2, B_2, C_2, A_1, and C_1, can be made to take care of the interaction between the cylinders. First, we equate σ_r along the common boundary $r = a$,

$$A_2 + (B_2/a^2) = A_1. \tag{A-7}$$

Then we equate to zero the resultant of the axial stress σ_z over a section z = constant of both cylinders:

$$C_2(b^2 - a^2) + C_1 a^2 = 0. \tag{A-8}$$

Again, since $r = b$ is a free boundary (the coefficient of thermal expansion for the plastic container is more or less the same with the one for the epoxy), σ_r vanishes there:

$$A_2 + (B_2/b^2) = 0. \tag{A-9}$$

Solving (A-7), (A-8), and (A-9) in terms of A_2 and C_2 yields

$$B_2 = -A_2 b^2,\ A_1 = A_2(1 - b^2/a^2),\ C_1 = C_2(1 - b^2/a^2). \tag{A-10}$$

Substituting in (A-1) and (A-2), we obtain for the stresses

$$\left.\begin{aligned}\sigma_r &= A_2(1 - b^2/r^2)\\ \sigma_\theta &= A_2(1 + b^2/r^2)\\ \sigma_z &= C_2\end{aligned}\right\} \text{ for outer cylinder} \quad \text{(A-11)}$$

$$\left.\begin{aligned}\sigma_r &= \sigma_\theta = A_2(1 - b^2/a^2)\\ \sigma_z &= C_2(1 - b^2/a^2)\end{aligned}\right\} \text{ for inner cylinder} \quad \text{(A-12)}$$

To determine the two remaining constants A_2 and C_2, we bring in the fact that along $r = a$, the displacements are the same. Equating ϵ_z for both cylinders (ϵ_z is independent of r), we obtain from (A-5) and the corresponding equation for the inner cylinder

$$(-A_2 2\nu_2/E_2) + (C_2/E_2) - (-A_2 2\nu_1/E_1 + C_1/E_1) = (\alpha_1 - \alpha_2)(T - T_0),$$

and, substituting from (A-10),

$$2A_2[\nu_2/E_2 - (1 - b^2/a^2)\nu_1/E_1] - C_2[1/E_2 - (1 - b^2/a^2)/E_1]$$
$$= (\alpha_2 - \alpha_1)(T - T_0). \quad \text{(A-13)}$$

To equate u at $r = a$, it suffices to equate ϵ_θ there; its value is given by the second of eqs. (A-5) for the outer cylinder and (omitting B and changing subscripts) by

$$E_1\epsilon_\theta = A_1(1 - \nu_1) - \nu_1 C_1 + E_1\alpha_1(T - T_0) \quad \text{(A-14)}$$

for the inner cylinder. Utilizing (A-10), we transform the resulting equations into

$$A_2[(1 - \nu_1)(1 - b^2/a^2)/E_1 + \nu_2(1 - b^2/a^2)/E_2 - (1 + b^2/a^2)/E_2]$$
$$- C_2[\nu_1(1 - b^2/a^2)/E_1 - \nu_2/E_2] = (\alpha_2 - \alpha_1)(T - T_0). \quad \text{(A-15)}$$

Solving (A-13) and (A-15) for A_2 and C_2 yields

$$A_2 = -(\alpha_2 - \alpha_1)(T - T_0)[(1 + \nu_1)(1 - b^2/a^2)/E_1 - (1 + \nu_2)/E_2]/D$$

$$C_2 = -(\alpha_2 - \alpha_1)(T - T_0)[(1 + \nu_1)(1 - b^2/a^2)/E_1 - (1 + \nu_2)(1 + b^2/a^2)/E_2]/D$$

where

$$D = \begin{vmatrix} 2\nu_2/E_2 - (1 - b^2/a^2)2\nu_1/E_1 & -1/E_2 + (1 - b^2/a^2)/E_1 \\ (1 - \nu_1)(1 - b^2/a^2)/E_1 + \nu_2(1 - b^2/a^2)/E_2 - (1 + b^2/a^2)/E_2 & \nu_2/E_2 - \nu_1(1 - b^2/a^2)/E_1 \end{vmatrix}.$$

This completes the determination of the stresses.

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